The Space-Time CE/SE Method for Solving Maxwell's Equations in Time-Domain

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Abstract

An innovative finite-volume-type numerical method named as the space-time conservation element and solution element (CE/SE) method is applied to solve time-dependent Maxwell's equations in this paper. Test problems of electromagnetics scattering and antenna radiation are solved for validations. Numerical results are presented and compared with the analytical solutions, showing very good agreements.

1. Introduction

The most popularly used numerical method for solving the time-dependent Maxwell's equations are the finite-difference time domain (FDTD) method originally introduced by Yee [1], and the finite-volume time-domain (FVTD) method developed by Shankar et al. [2] and Shang [3]. The space-time CE/SE method was developed by Chang [4] and has been successfully applied in computational fluid dynamics (CFD) and computational aeroacoustics (CAA) [5]. It is initiated here to apply this method for solving Maxwell's equations. Its salient properties are summarized briefly as follows. First, both local and global flux conservations are enforced in space and time instead of in space only. Second, all the dependent variables and their spatial derivatives are considered as individual unknowns to be solved for simultaneously at each grid point. Third, every CE/SE scheme is based upon a non-dissipative scheme with addition of fully controllable numerical dissipation. This results in very low numerical dissipation. Fourth, it can use both structured and unstructured meshes in one single algorithm to handle complex geometries, and it has the most compact stencil, this leads to efficient parallel computing and easy implementation of boundary conditions.

The same procedures used to construct numerical schemes for the Euler equations in [6] are applied to Maxwell's equation here. Test problems of electromagnetics scattering and antenna radiation are solved for validations. Numerical results are presented and compared with the analytical solutions in the following.

2. Numerical Results

The first example is about the scattering field of transverse magnetic(TM) waves due to a perfect conducting circular cylinder, which is depicted in Fig. 1. The radius of the cylinder is denoted as a, and the incoming wave has electric intensity E_z and magnetic intensity H_x, H_y components that are described as

$$E_z = \exp(i(\omega t - kx)), \quad H_x = 0, \quad H_y = -\exp(i(\omega t - kx)) \tag{1}$$

where k is the wave number and ω is the frequency. Three different wavenumbers ($ka = 1.0, 5.0,\ 10.0$) are considered here. The obtained solutions of the scattering field are plotted in Figs. 2–4 with the analytical solutions. The distribution of E_z, H_x, and H_y on the cylinder surface at one time instance plotted in Fig. 2 shows good agreement for the three wavenumbers. The normalized surface current plotted in Fig. 3 agrees well with the analytical solution for $ka = 1.0$ and 5.0, while some discrepancies at the trailing edge of the circular cylinder are observed, which is expected to be improved by using a finer mesh on the cylinder surface. The radar cross section (RCS) in dB is plotted in Fig. 4, showing good agreements. The definitions of the normalized surface current and radar cross section (RCS) in dB are referred to in [2]. The mesh used here is 91x61, 121x121, and 241x161 for $ka = 1.0, 5.0$ and 10.0, respectively.

The second example regards the radiation of a 300MHz half-wavelength dipole antenna. The computational domain is a cubic box formed by $-0.5 \leq x, y, z \leq 0.5$, in which the half-wavelength antenna is located at $x = y = 0, -0.25 \leq z \leq 0.25$. The thickness of the antenna is not considered here. And the current density $\vec{J} = (J_x, J_y, J_z)$ along the antenna surface is defined as:

$$J_x = 0, \quad J_y = 0, \quad J_z = -\cos(\omega t)\sin(k(0.25 - |z|))/\sin(k/4) \tag{2}$$

A uniform 21x21x21 mesh is used in the simulation. The directivity pattern of the half-wavelength dipole is computed through the near-to-far-field transformation. The computed result of the directivity pattern in 2D is plotted in Fig. 5 with the analytical solution, showing a good agreement. The computed 3D directivity pattern is shown in Fig. 6.

3. Conclusion

The space-time CE/SE method for solving Maxwell's equation in time-domain has been presented. Numerical validations show that some preliminary success has been achieved. More tests need to be done to show the potential of the CE/SE method for solving more complicated problems, such as those involving anisotropic media, dispersive media, and bodies with complex geometries.

References

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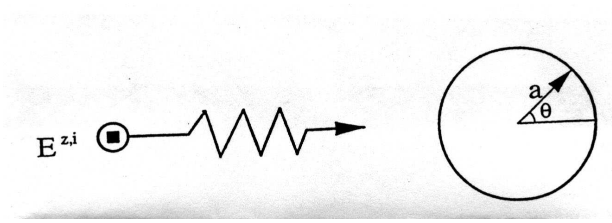
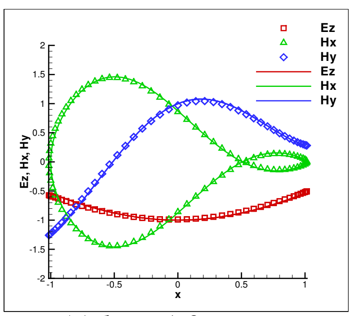
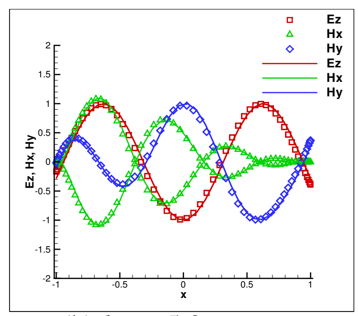


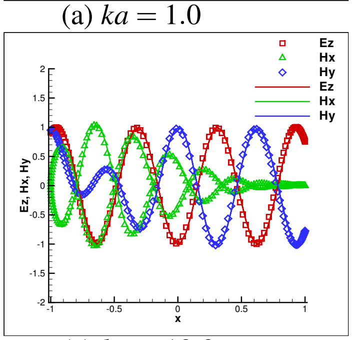
Figure 1: Scattering by a circular cylinder.



(a) $ka = 1.0$

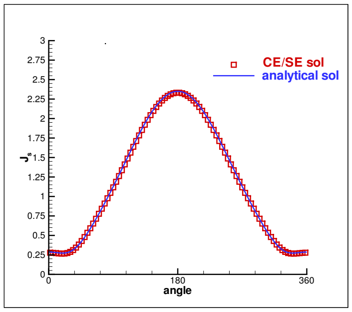


(b) $ka = 5.0$

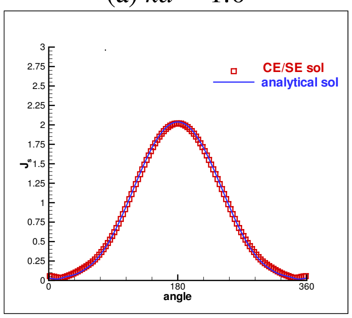


(c) $ka = 10.0$

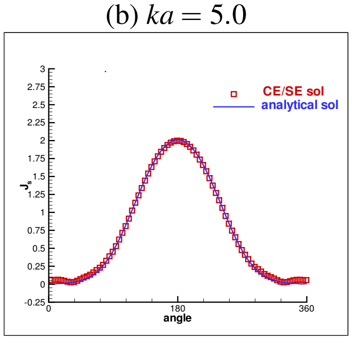
Figure 2: Solution of E_z, H_x, H_y on the cylinder surface at one time level.



(a) $ka = 1.0$

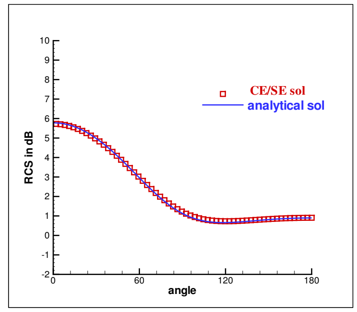


(b) $ka = 5.0$

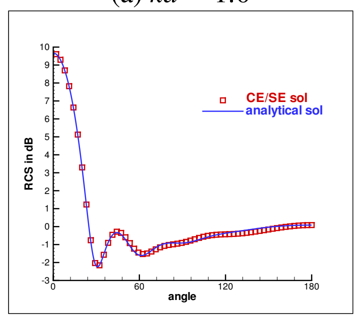


(c) $ka = 10.0$

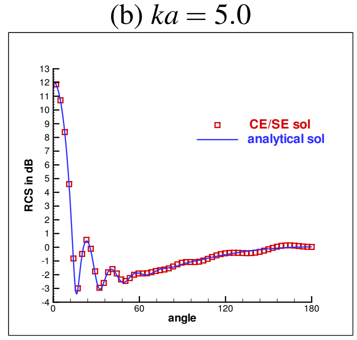
Figure 3: Normalized surface current for the circular cylinder.



(a) $ka = 1.0$



(b) $ka = 5.0$



(c) $ka = 10.0$

Figure 4: RCS for the circular cylinder.

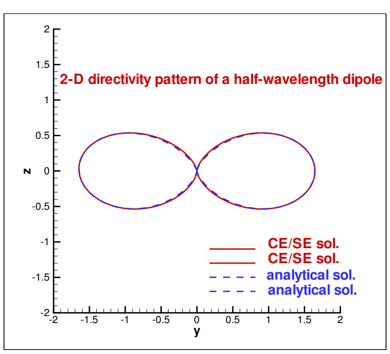


Figure 5: The 2D directivity pattern of a half-wavelength dipole.

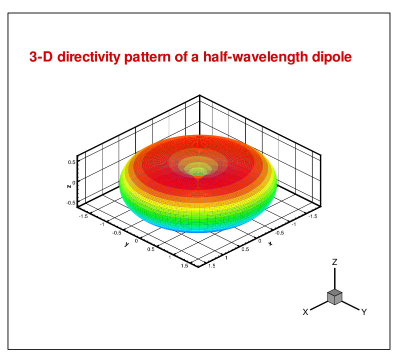


Figure 6: The 3D directivity pattern of a half-wavelength dipole.